Generalized Statistical Traffic Assignment (GESTA)

Methodology, Properties and Variance Analysis

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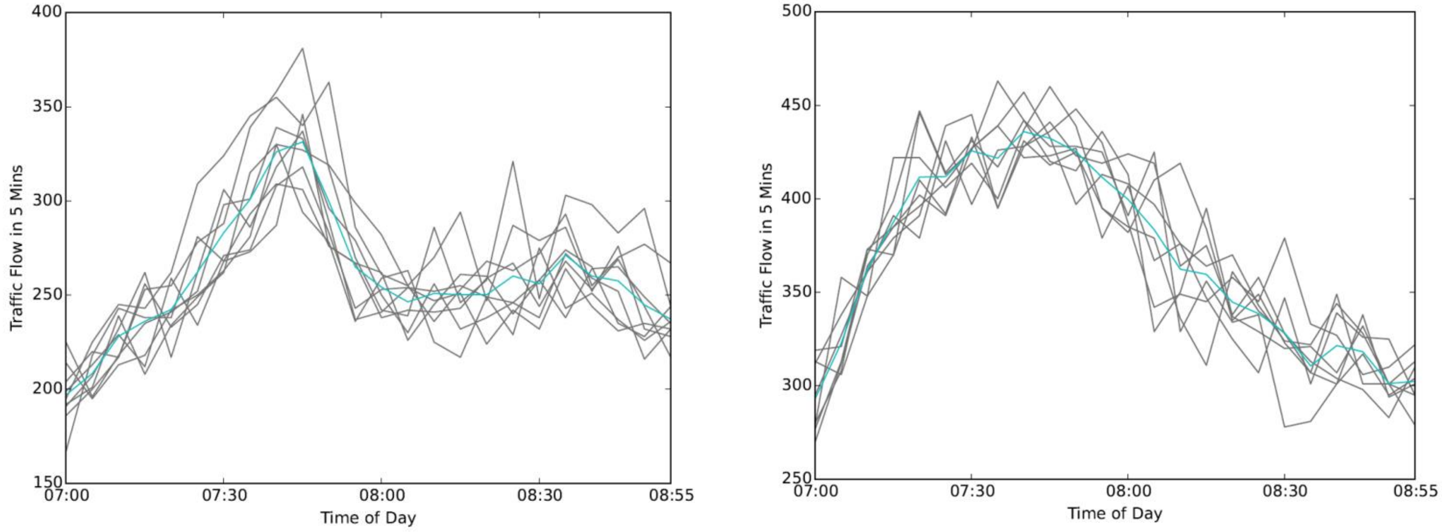
Background

- ❑ Classical traffic assignment plays a pivotal role in planning and operations

$$\phi: q \longmapsto x, f$$

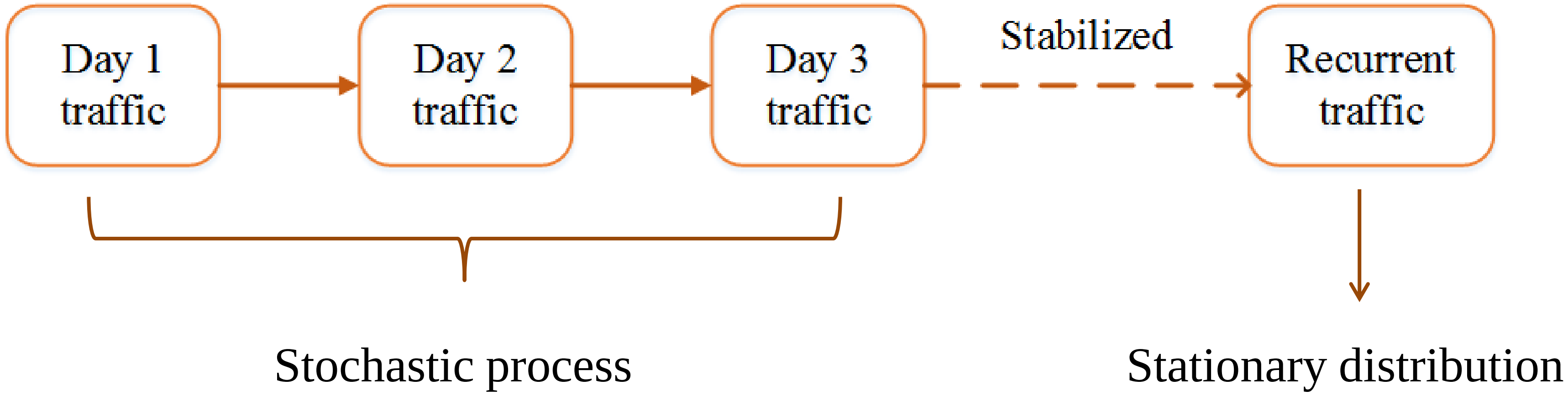


Daily time-varying traffic counts on SR41 SB and NB

- ❑ Statistical traffic assignment explores the statistical features of recurrent flow patterns

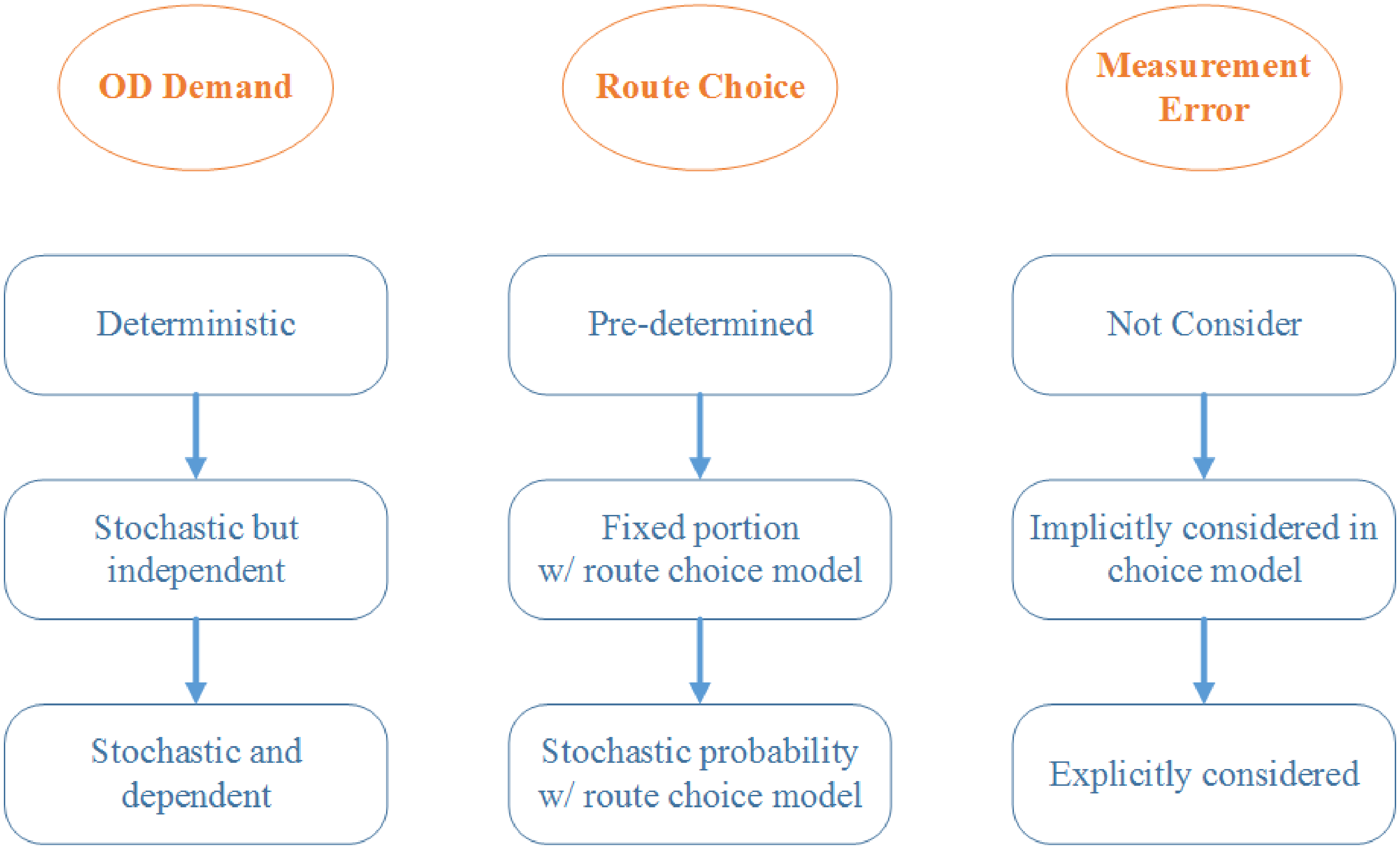
$$\phi: Q \longmapsto X, F$$

Stochastic process vs. Stationary distribution



Literature review

- ❑ Network variation ≈ Flow variation
- ❑ Flow variation comes from different sources



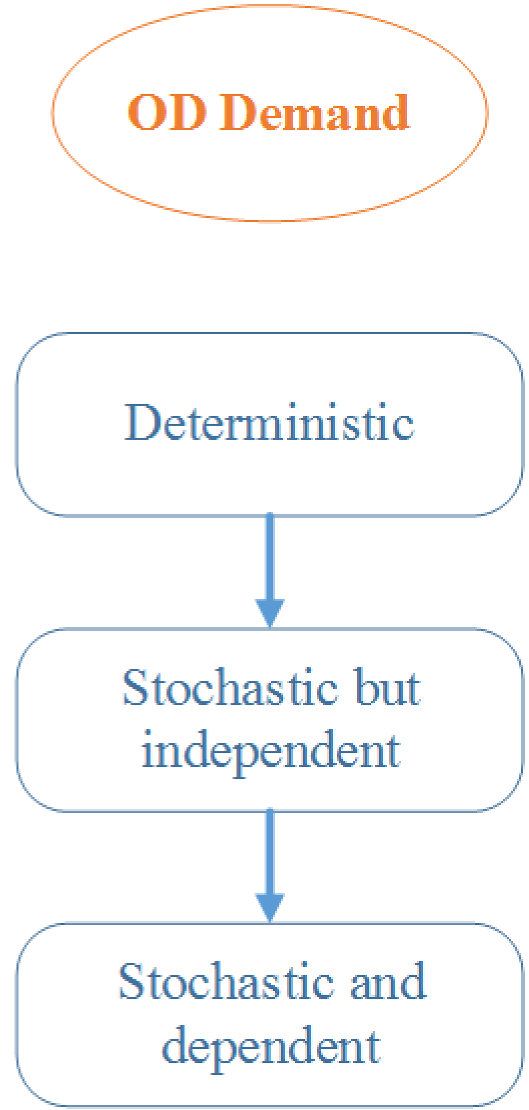
OD demand variation

- ❑ OD was first considered as a deterministic value
- ❑ Variance of OD was modelled but the covariance was not (e.g. Poisson distribution)
- ❑ Both variance and covariance was modelled (e.g. MVN)

<u>Example:</u>

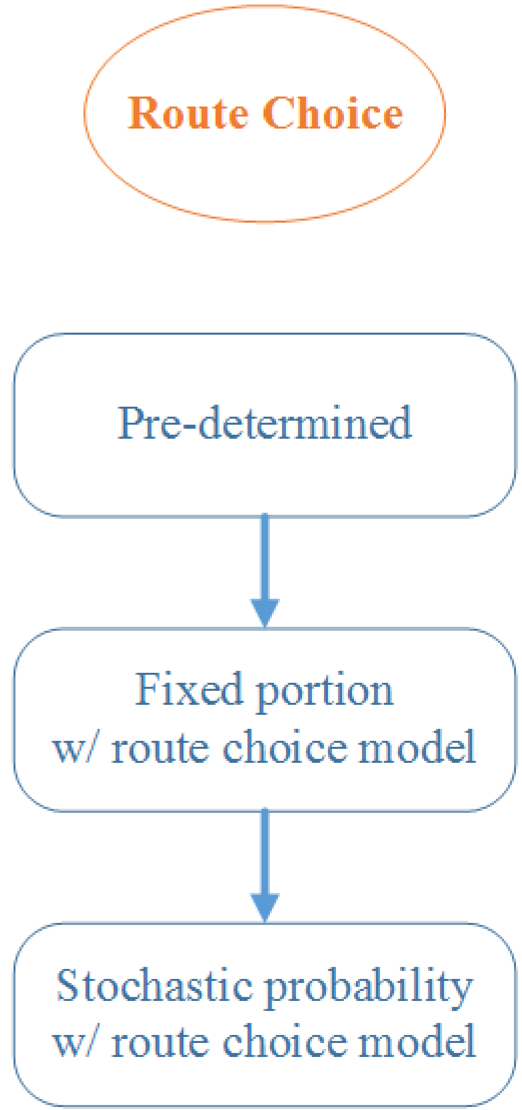
$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \Rightarrow Q_i \sim Pois(\lambda_i) \Rightarrow Q \sim MVN(q, \Sigma_q)$$

MVN: Multi-variate Normal Distribution



Route choice variation

- Understand the route choice probability p
- In uncongested networks, the route choice can be pre-determined
- The fixed proportion p that travelers will select certain routes
- The probability p for an arbitrary traveller to choose certain route



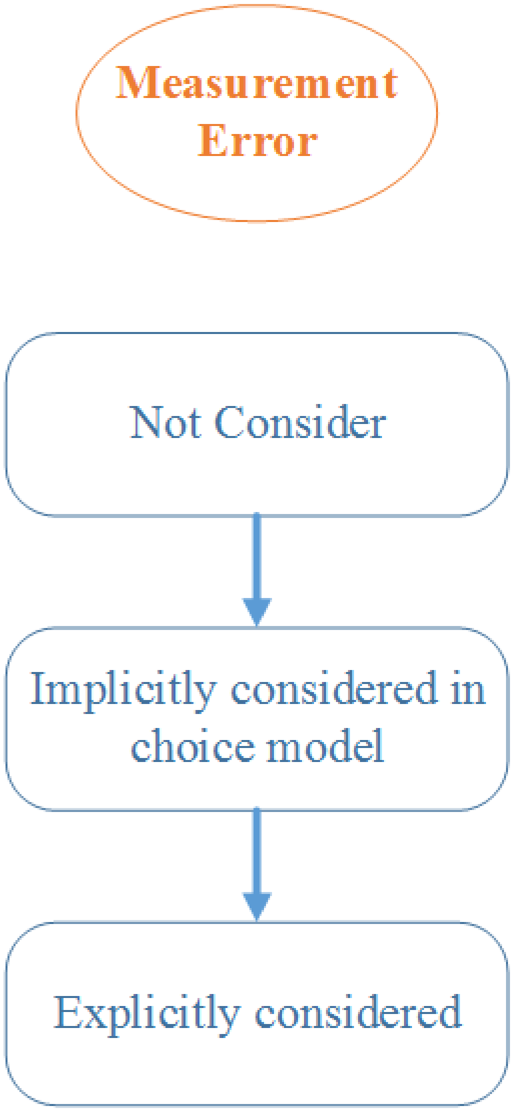
<u>Example:</u>

100 travelers, 2 routes, $\mathrm{p}_1 = 0.3, p_2 = 0.7$

$$f_1 = 30 \quad \Rightarrow \quad F_1 \sim BN(100, 0.3)$$

Measurement error

- ❑ Measurement error is different from perception error, it exists physically
- ❑ Choice model may combine the perception error and measurement error (e.g. logit/probit)
- ❑ Explicitly considering the measurement error helps to calibrate the model



Our study

OD demand	Route Choice	Measurement Error	Model
Deterministic	Pre-determined	Not consider	Trivial case
Deterministic	Fixed portion w/ route choice model	Implicitly considered in choice model	Logit/Probit model
Stochastic but independent	Pre-determined	Not consider	Vardi 1996, Hazelton 2008, Parry & Hazelton 2012, Hazelton et al. 2015
Stochastic but independent	Stochastic probability w/ route choice model	Implicitly considered in choice model	Nakayama &Watling 2014, Castillo et al. 2014
Stochastic but independent	Fixed portion w/ stochastic route choice model	Implicitly considered in choice model	Nakayama & ichi Takayama 2006, Nakayama & Takayama 2003
Stochastic and dependent	Fixed portion w/ route choice model	Implicitly considered in choice model	Shao, Lam & Tam 2006, Lam et al. 2008, Shao et al. 2014, 2015
Stochastic and dependent	Stochastic probability w/ route choice model	Explicitly considered	This study

Model

- OD
  - MVN: $Q \sim N(q, \Sigma_q)$
  - Scalable, stable and consistent, Castillo et al. (2014)
- Route choice
  - p is deterministic for a recurrent traffic network
  - A generalized probability function: $p = \psi(C; \Theta)$
- Path flow
  - Path flow follows multinomial distribution
  - $F_{rs} | Q_{rs} \sim MN(Q_{rs}, p_{rs})$
  - Normal approximation
- Link flow
  - $X = \Delta F$, where Δ is the path/link incidence matrix
- Path cost
  - A generalized path cost function: $C \sim t(X; \Theta)$
  - Normal approximation

Model: Hierarchy

Level 1:

$$X_m \sim N(X + e, \Sigma_x + \Sigma_e)$$ Measurement error

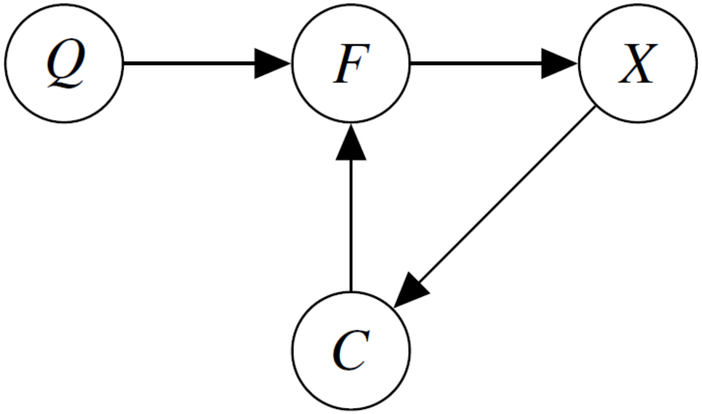
Level 2:

$$X \sim N(\Delta p Q, \Sigma_x)$$
$$F \sim N(pQ, \Sigma_f)$$ Route choice variation

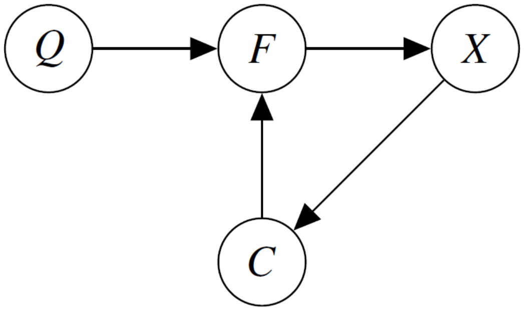
Level 3:

$$Q \sim N(q, \Sigma_q)$$ OD variation

- ❑ Level i is conditional on level i+1
- ❑ Each level reflects one single source of the link flow variation



Model: Property



- Existence and consistence of the solution
  - Fixed point problem
  - If the path cost function is continuous, the solution $(x, \Sigma_x, f, \Sigma_f, p)$ exists.
  - Once the solution $(x, \Sigma_x, f, \Sigma_f, p)$ is determined, then it's consistent.
- Reducible
  - Model can be reduced to existing statistical assignment models
- Data driven
  - Friendly to model learning techniques for large scale networks
  - Dimension reduction, sparse regularization, model selection

Model: Application

- ❑ Marginal distribution

Marginal distribution of X_m can be written as:

$$X_m \sim N(x_m, \boxed{\Delta p \Sigma_q p^T \Delta^T} + \boxed{\Delta \Sigma_{f|q} \Delta^T} + \boxed{\Sigma_e})$$

Three matrices represent the variance from OD, route choice and measurement error separately.

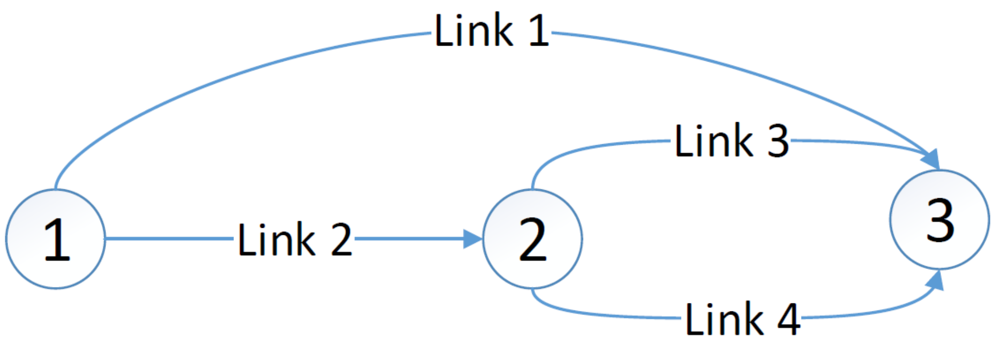
- ❑ Variance ratio
  - To measure the portion of each source of variance
  - To help to reduce the system variance
  - Trace norm, nuclear norm
- ❑ MANOVA
  - Hypnosis testing
  - Calculate confidence interval (CI) of link flows

Solution algorithms

- ❑ Path based: K-shortest paths
- ❑ Alternating optimization: may not converge
- ❑ MSA based algorithm: may not converge to global optimum
- ❑ Computational complexity: quadratic to # of paths

Experiment

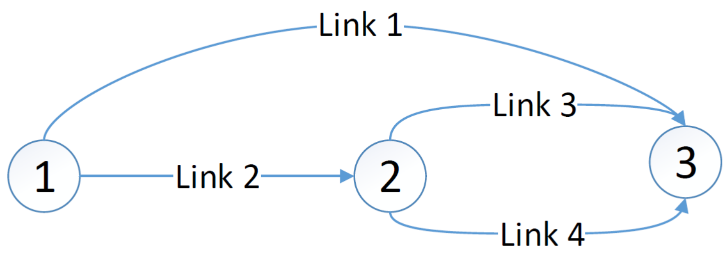
❑ Settings



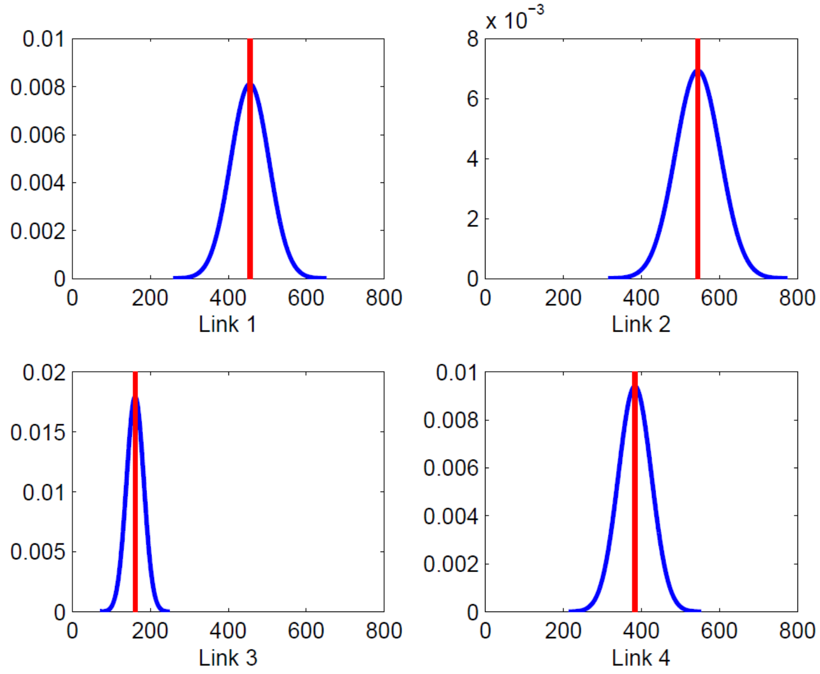
- OD: 1 -> 3, $q = 1000$, $\sigma^2 = 10000$
- Route choice model: Probit + Multinomial
- Link cost function: BPR function
- Measurement error: $e = 0$, $\sigma_e^2 = 100$

Experiment

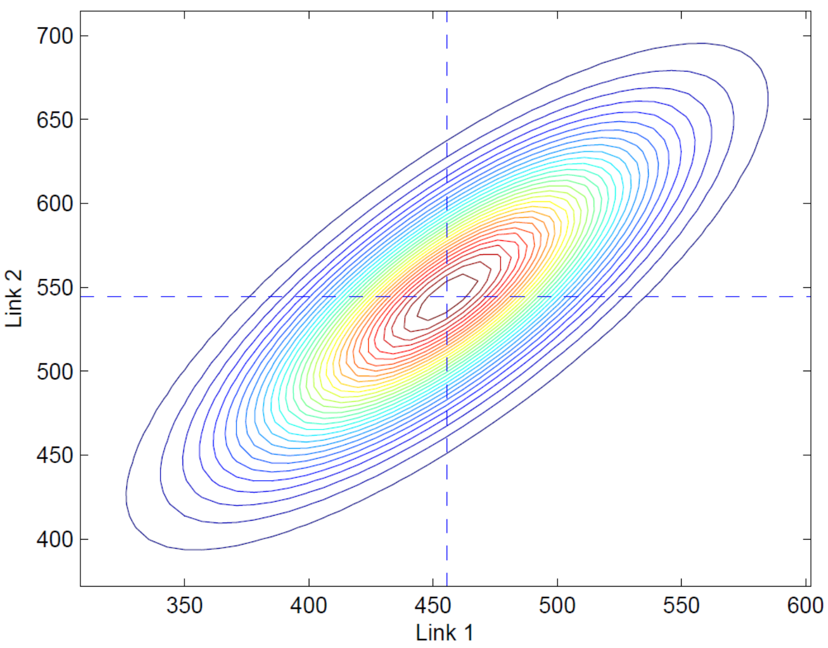
- ❑ Basic results



Network condition

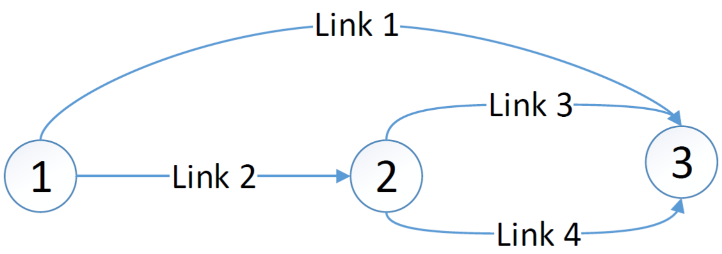


Confidence interval

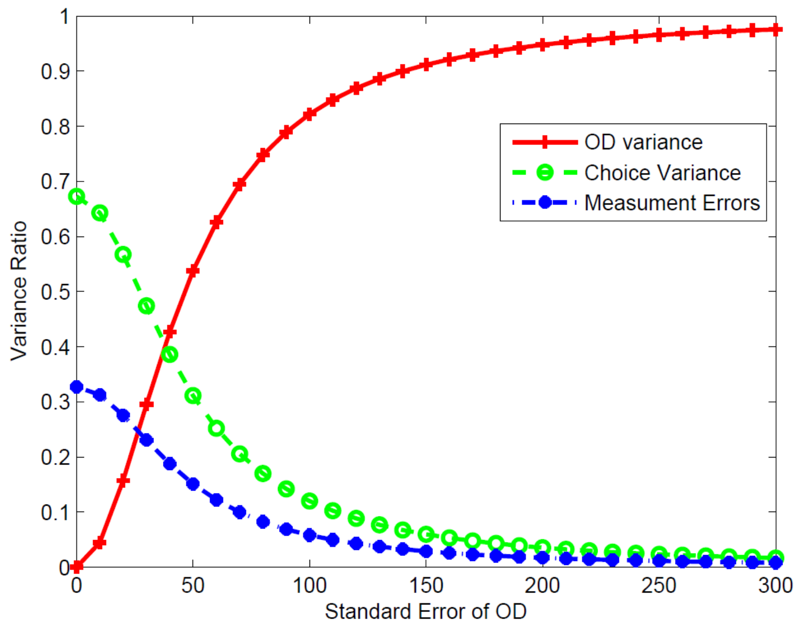


Experiment

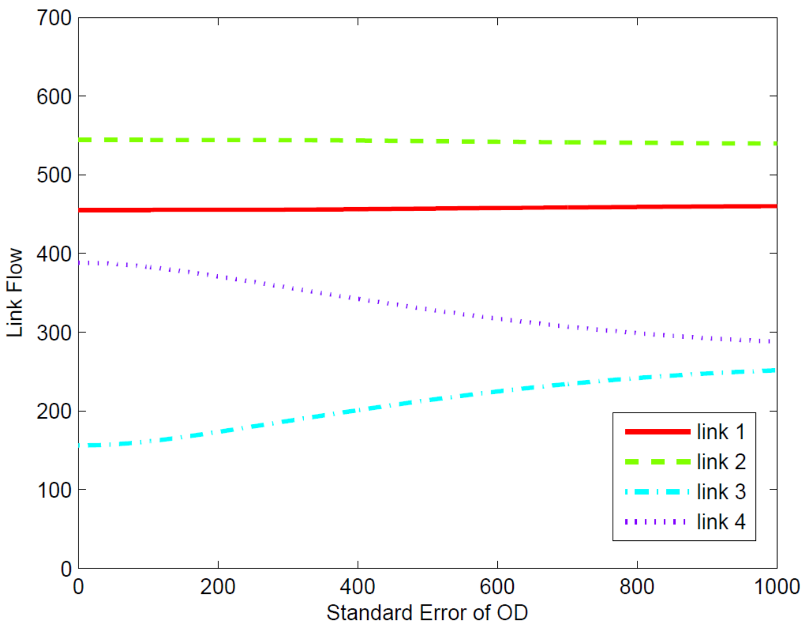
- ❑ Varying OD variance



Change of variance ratio



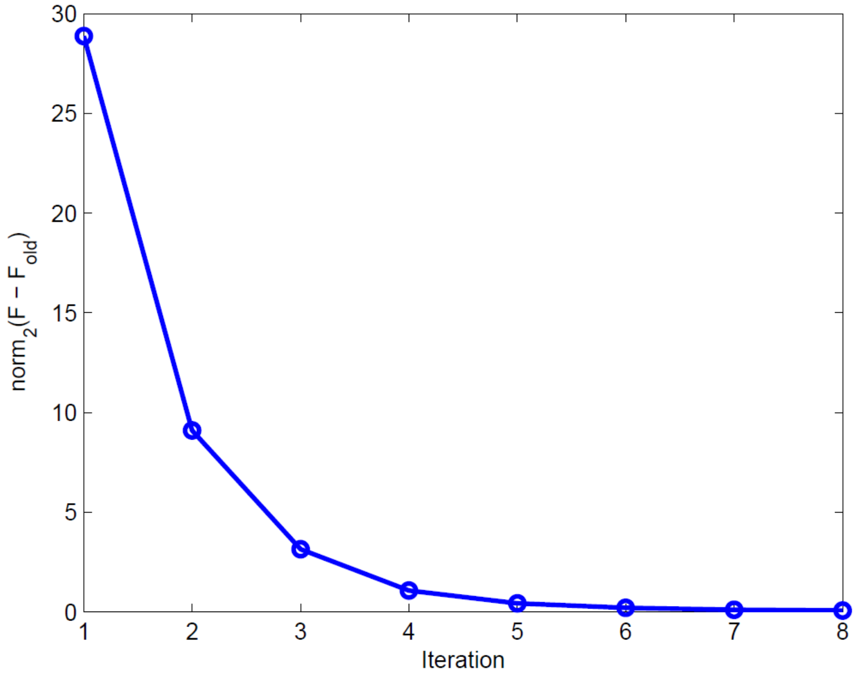
Change of link flow

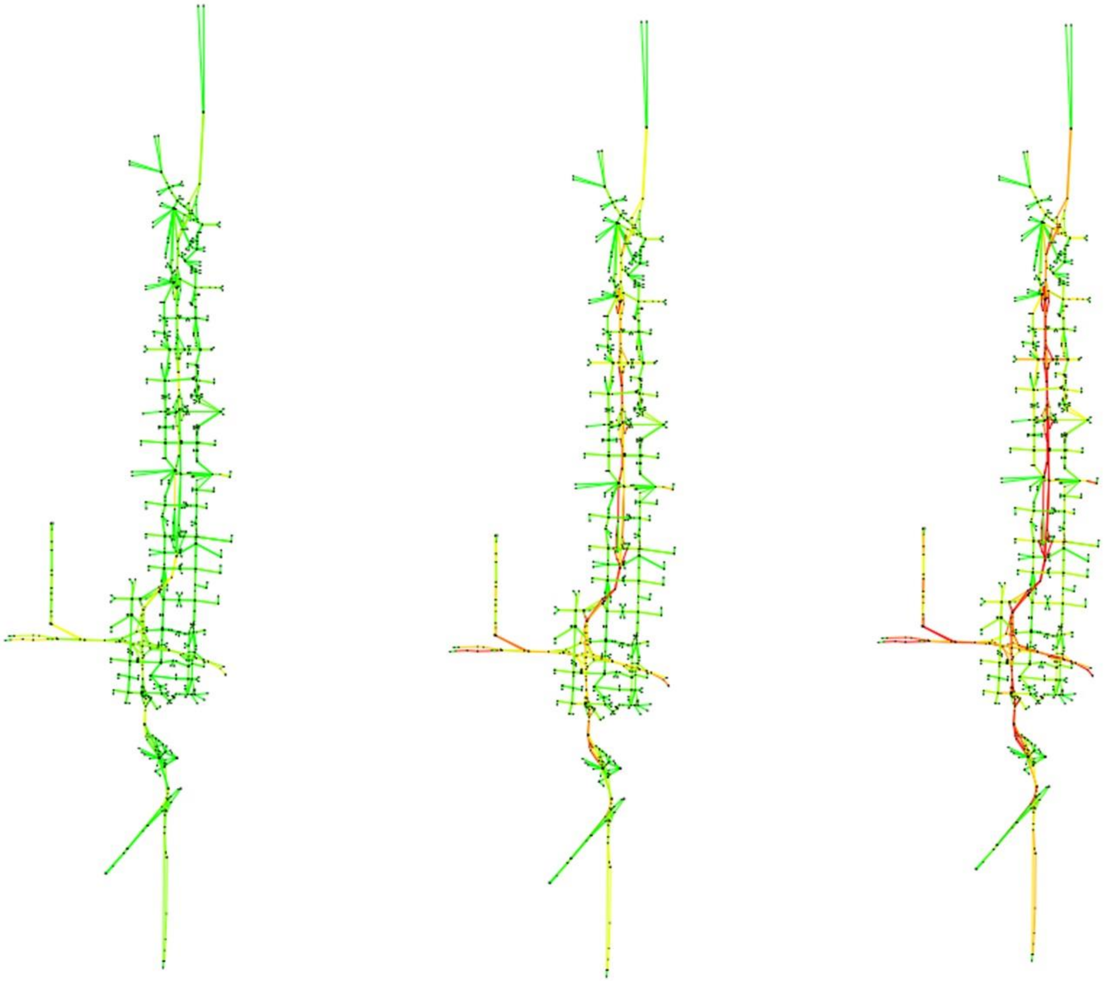


Variance ratio: trace based

Large Scale Network

- Settings
  - SR41 freeway network: 2413 links and 7110 OD pairs
  - OD demands were carefully calibrated
  - Assume the O-D demand variance is 20% of its mean
- Results
  - Terminated in 9 iterations and 467 seconds





- Using Rectangle CI approximation.
- Red represents volume/capacity>1, and green represents volume/capacity=0, other colors are smoothly transitioned from green to red as volume/capacity increases from 0 to 1)

Highlights

- ❑ To generalize and extend previous studies on statistical traffic assignment models
- ❑ To prove the existence and consistence of the network equilibrium
- ❑ To demonstrate the analytical application of GESTA: marginal distribution, variance decomposition, MANOVA
- ❑ To test the model on a large-scale networks and prove its computational efficiency

Future work

- ❑ Sensitivity analysis of GESTA
- ❑ Calibration of GESTA (OD estimation)
- ❑ Dynamic-GESTA

Thanks!

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