Estimating multi-class dynamic origin-destination demand through a forward-backward algorithm on the computational graph

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Abstract

The multi-class dynamic origin-destination (OD) demand plays a central role in the transportation network modeling. Due to the lack of studies focusing on the multi-class dynamic OD demand, this paper presents a solution framework for multi-class dynamic OD demand estimation (MCDODE) on large-scale networks. The proposed framework is built on a computational graph with tensor representations of all the variables involved in the MCDODE formulation. A novel forward-backward algorithm is proposed to efficiently solve the MCDODE formulation on the computational graph. In the forward-backward algorithm, a tree-based cumulative curve is adopted to evaluate the gradient of OD demand. The proposed framework is examined on a small network as well as a real-world large-scale network. The experiment results are compelling, satisfactory and computationally plausible.

Background

- The dynamic OD estimation (DODE) problem has been extensively studied over the past few decades.
- However, there is a lack of multi-class dynamic OD demand estimation method that can be applied to large-scale networks with real-world data.
- MCDOD can help the policymakers understand the impact of each vehicle class to the roads, and hence the traffic management and operation policy for a specific vehicle class can be studied.

Modeling multi-class dynamic traffic flow

- The relation between OD flow and path flow

$$x_{ai}^{h_2} = \sum_{rs\in K_q}\sum_{k\in K_{rs}}\sum_{h_1\in H} \rho_{rsi}^{ka}(h_1,h_2) f_{rsi}^{kh_1}$$

- The relation between path flow and link flow

$$f_{rsi}^{kh_1} = p_{rsi}^{kh_1} q_{rsi}^{h_1}$$

- The relation between link flow and observed flow

$$y_b = \sum_{i\in D}\sum_{a\in A}\sum_{h_2\in H} L_{ai}^{bh_2} x_{ai}^{h_2}$$

- Combining above relation, we have

$$y_b = \sum_{i\in D}\sum_{a\in A}\sum_{h_2\in H} L_{ai}^{bh_2}\left(\sum_{rs\in K_q}\sum_{k\in K_{rs}}\sum_{h_1\in H} \rho_{rsi}^{ka}(h_1,h_2) p_{rsi}^{kh_1} q_{rsi}^{h_1}\right)$$

The Computational graph

- The basic formulation for multi-class dynamic origin-destination estimation (MCDODE).

$$\begin{aligned}
\min_{\{\mathbf{q}_i\}_i} \quad & \left(\left\|\mathbf{y} - \sum_{i\in D}\mathbf{L}_i\rho_i\mathbf{p}_i\mathbf{q}_i\right\|_2^2\right) \\
\text{s.t.} \quad \{\mathbf{c}_i,\rho_i\}_i &= \mathtt{DNL}(\mathbf{f}) \\
\mathbf{p}_i &= \Psi_i(\{\mathbf{c}_i\}_i) \quad \forall i\in D \\
\mathbf{q}_i &\geq 0 \quad \forall i \in D
\end{aligned}$$

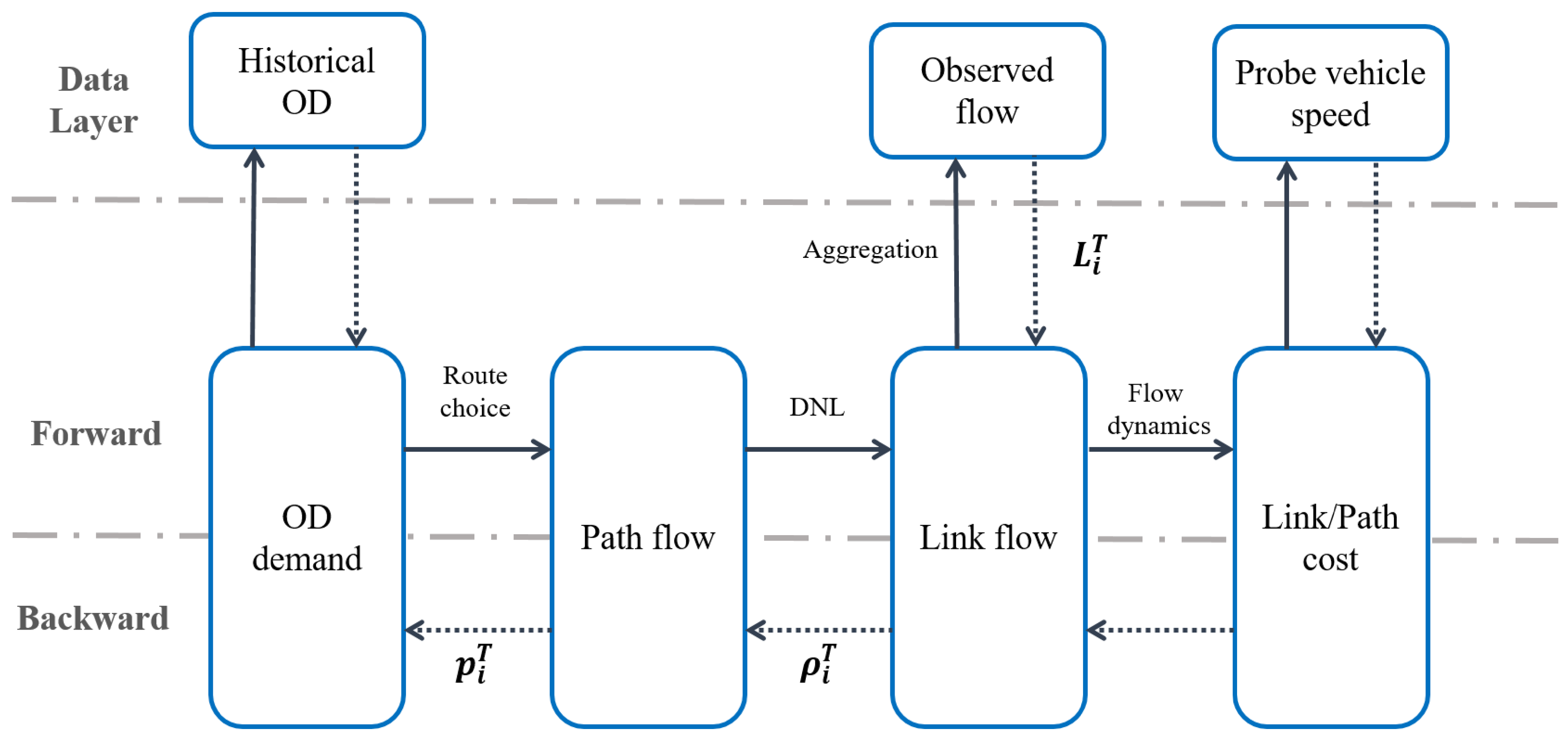
- **Forward iteration** solves the dynamic traffic assignment problem

$$\begin{aligned}
L &= \|\mathbf{y}' - \mathbf{y}\|_2^2 \\
\mathbf{y} &= \textstyle\sum_{i\in D}\mathbf{L}_i\mathbf{x}_i \\
\mathbf{x}_i &= \rho_i\mathbf{f}_i \\
\mathbf{f}_i &= \mathbf{p}_i\mathbf{q}_i
\end{aligned}$$

- **Backward iteration** uses the back propagation to derive the gradient of OD demand

$$\begin{aligned}
\frac{\partial L}{\partial \mathbf{y}} &= 2\left(\mathbf{y} - \textstyle\sum_{i'\in D}\mathbf{L}_{i'}\rho_{i'}\mathbf{p}_{i'}\mathbf{q}_{i'}\right) \\
\frac{\partial L}{\partial \mathbf{x}_i} &= -\mathbf{L}_i^T\frac{\partial L}{\partial \mathbf{y}} \\
\frac{\partial L}{\partial \mathbf{f}_i} &= \rho_i^T\frac{\partial L}{\partial \mathbf{x}_i} \\
\frac{\partial L}{\partial \mathbf{q}_i} &= \mathbf{p}_i^T\frac{\partial L}{\partial \mathbf{f}_i}
\end{aligned}$$

- **Framework of forward-backward algorithm**



- **All training techniques in the deep neural networks can be applied!**
  - ➢ GPU acceleration
  - ➢ Parallel computing (parameter server)
  - ➢ Adaptive gradient descent: Adam, Adadelta, Adagrad

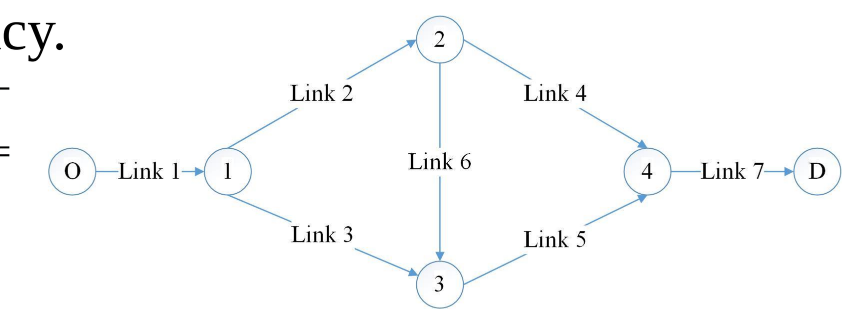
Numerical Experiments

- **A 7-link network**

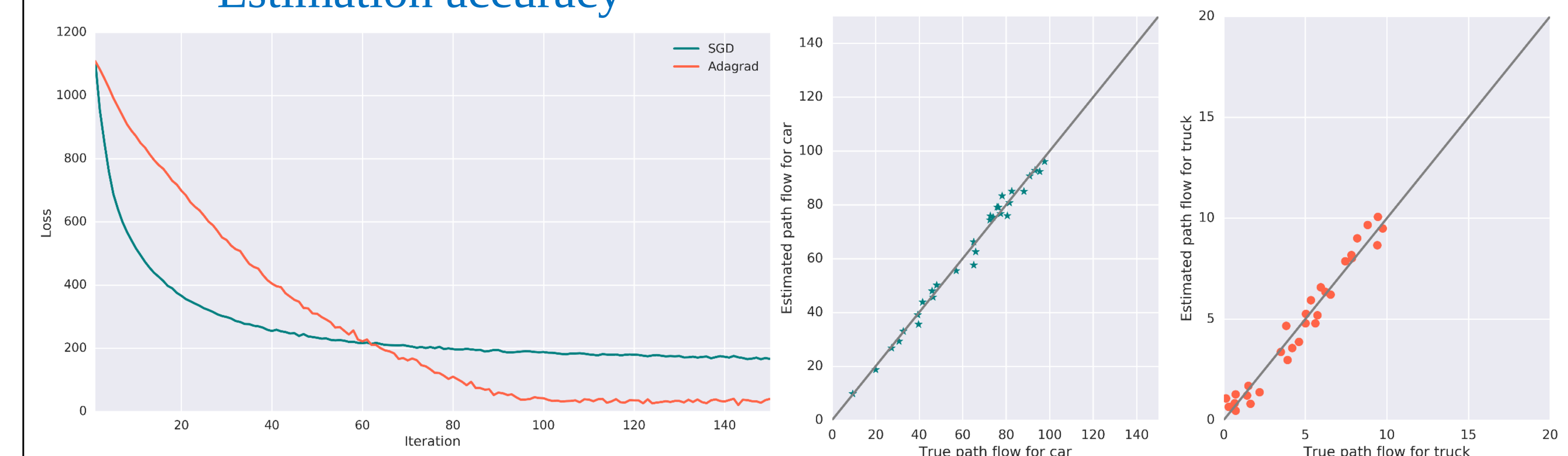
We construct the multi-class dynamic OD demand by random number generators and then treat it as the "true" OD demand. We use R-square between the "true" flow and estimated flow to measure the estimation accuracy.

	Car	Truck
Observed flow	0.9986	0.9716
Link flow	0.9959	0.9466
Path flow	0.9873	0.9681

Estimation accuracy

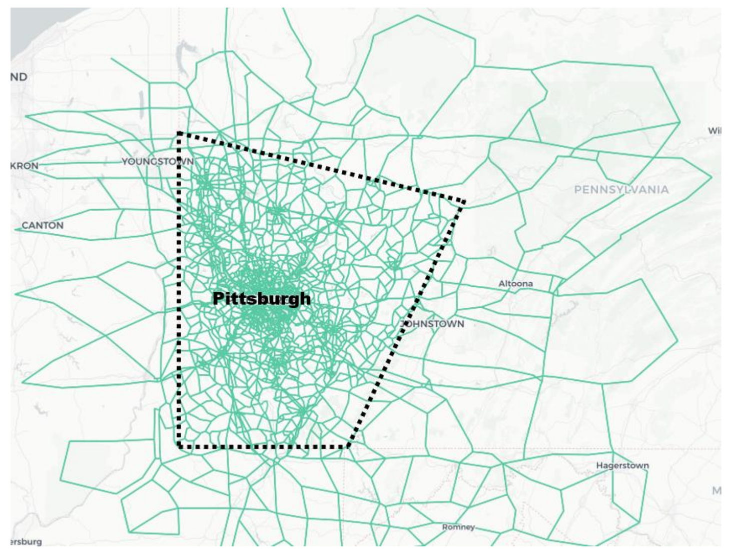


Network overview

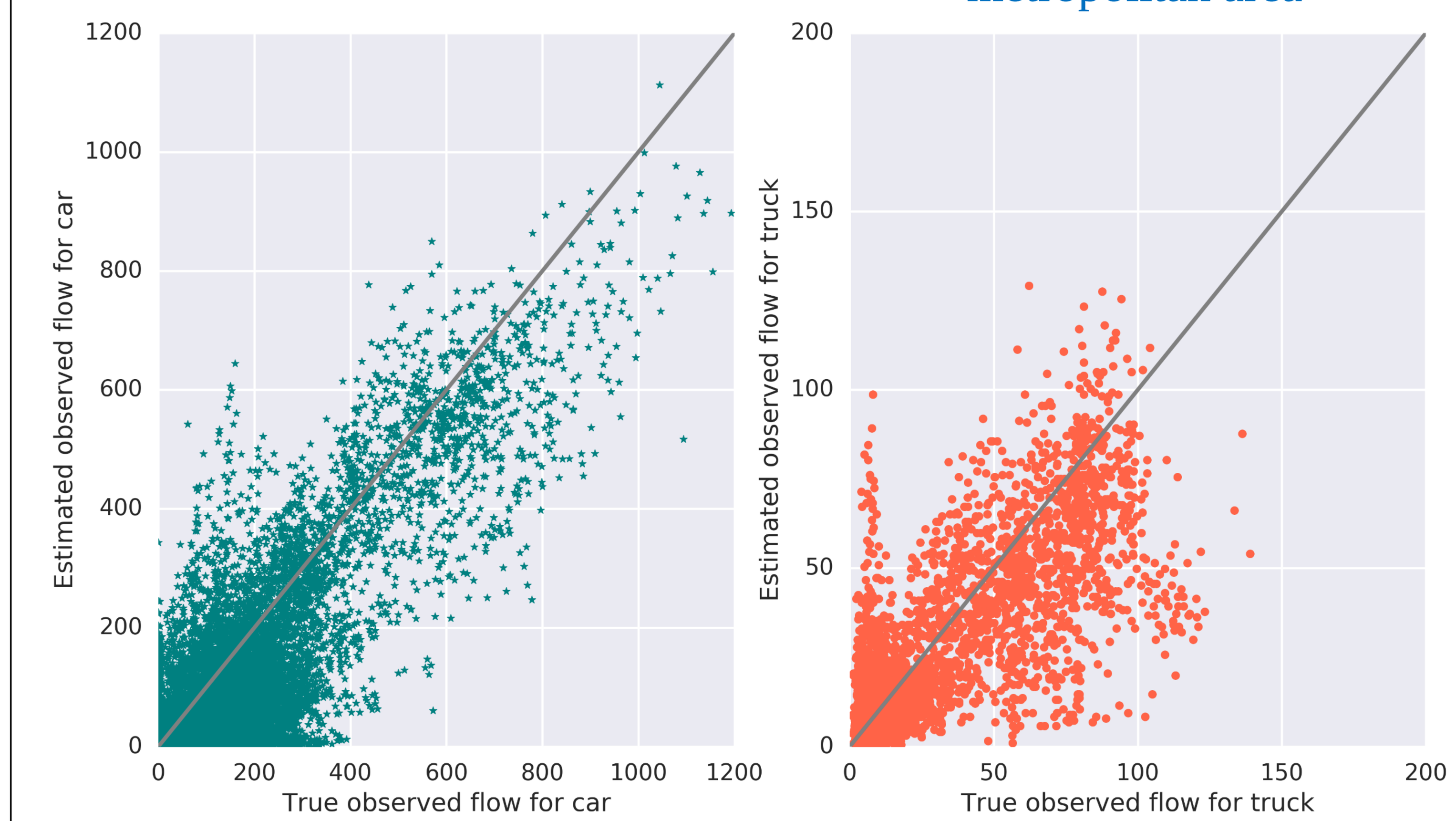


Convergence curve and estimated path flow

- **A large-scale network: Pittsburgh metropolitan area**
  - ➢ The network covers the ten counties of southwestern Pennsylvania region, with the Pittsburgh city in the center.
  - ➢ The network also consists of 16,110 links, 6,297 nodes and 283 origins/destinations.



Overview of Pittsburgh metropolitan area



Estimated and observed flow for cars and trucks (unit: vehicle/15mins)

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