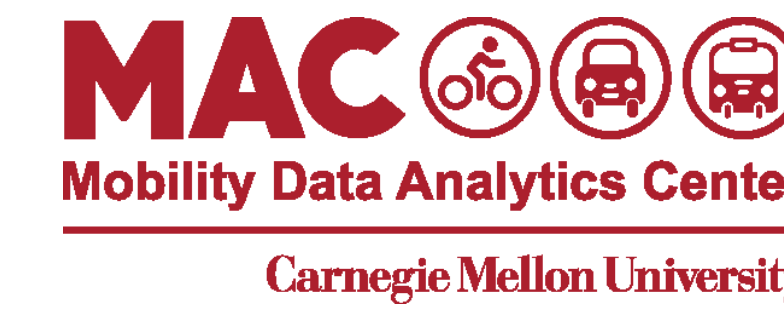


# On the Variance of Recurrent Traffic Flow for Statistical Traffic Assignment

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## Abstract

### Motivation:

The origin-destination (OD) demand is a critical input to system modeling in transportation planning and management. For decades, OD demand is deterministically modeled as it's translated into deterministic link/path flow and travel cost. Recent studies on transportation network uncertainty and reliability call for modeling the stochasticity of OD demand, namely its spatio-temporal correlation and variation.

### Objectives:

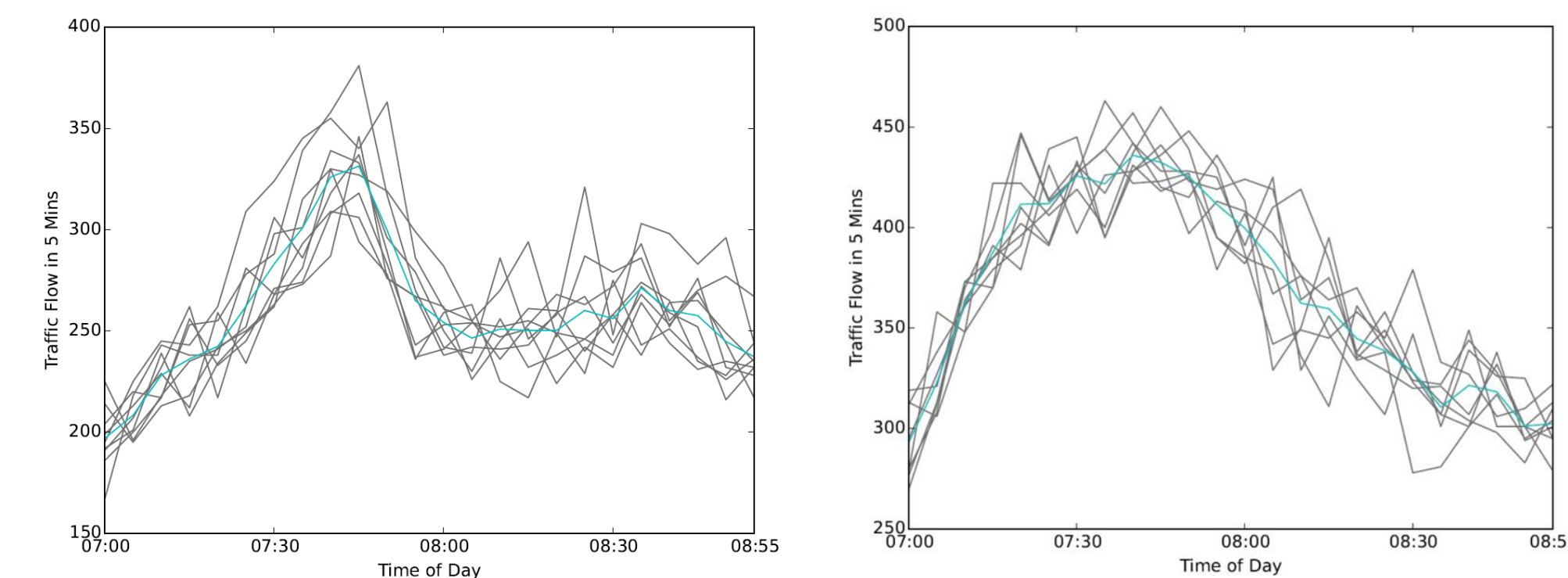
Build statistical relationship between traffic demand and network conditions (link/path flow, travel time).

## Background

Classical traffic assignment and OD estimation methods only model the averaged condition of the traffic condition.

$$\phi: q \mapsto x, f$$

Real world traffic data are coupled with temporal-spatio noise.



Daily time-varying traffic counts on SR41 SB and NB

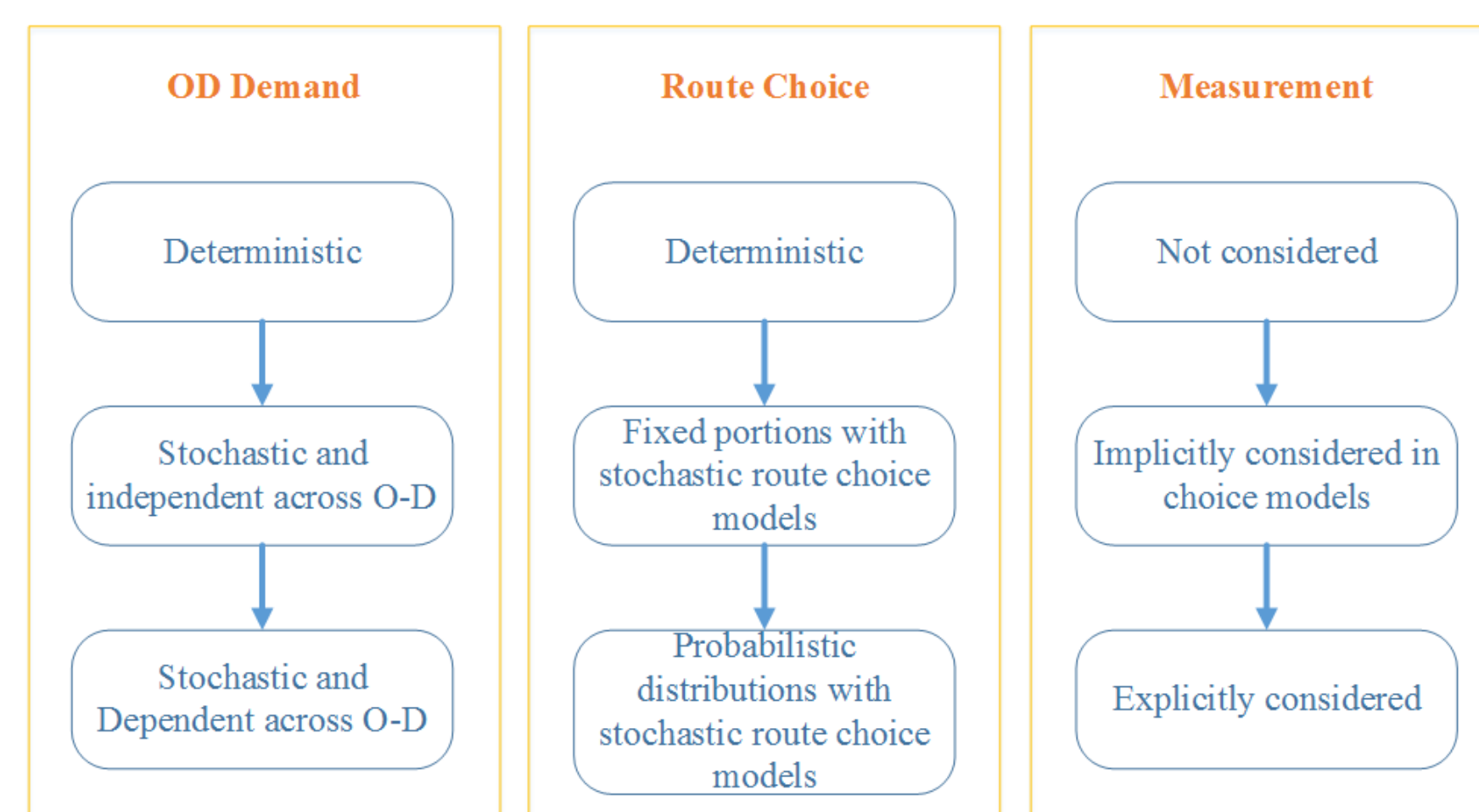
(Blue curve is the daily average. Each grey curve represents one day in May 2015)

Statistical traffic assignment explores the statistical features of recurrent flow patterns.

$$\phi: Q \mapsto X, F$$

Day to day traffic flow contains multidimensional variations.

### Day-to-day flow variations



Traffic data include road flow, travel speed and history OD (with noise).

## Model detail

### OD demand

- OD demands is represented by  $Q_{rs}$
- OD demands follow a rounded multivariate normal (MVN) distribution.

$$Q \sim N(q, \Sigma_q)$$

### Individual route choice probability

- Travelers select their route by a generalized route choice model.
- If Random Utility Model (RUM) is used, the route choice can be represented by

$$p = \Psi(c, \Sigma_c)$$

$$p_{rs,k} = \Pr[C_{rs,k} \leq \min_{i \neq k} C_{rs,i}]$$

### Path flow

- Each traveler independently makes identically distributed route choices on every day, which follows a multinomial distribution.

$$F_{rs} | Q_{rs} \sim MN(Q_{rs}, p_{rs})$$

- A norm approximation is adopted to find the analytical solution.

$$F_{rs} | Q_{rs} \sim N(Q_{rs}, p_{rs})$$

### Link flow and cost

- Link flow is a linear transformation from the path flow.
- Link cost is a defined as a continuous function, and can be approximated by Taylor expansion.

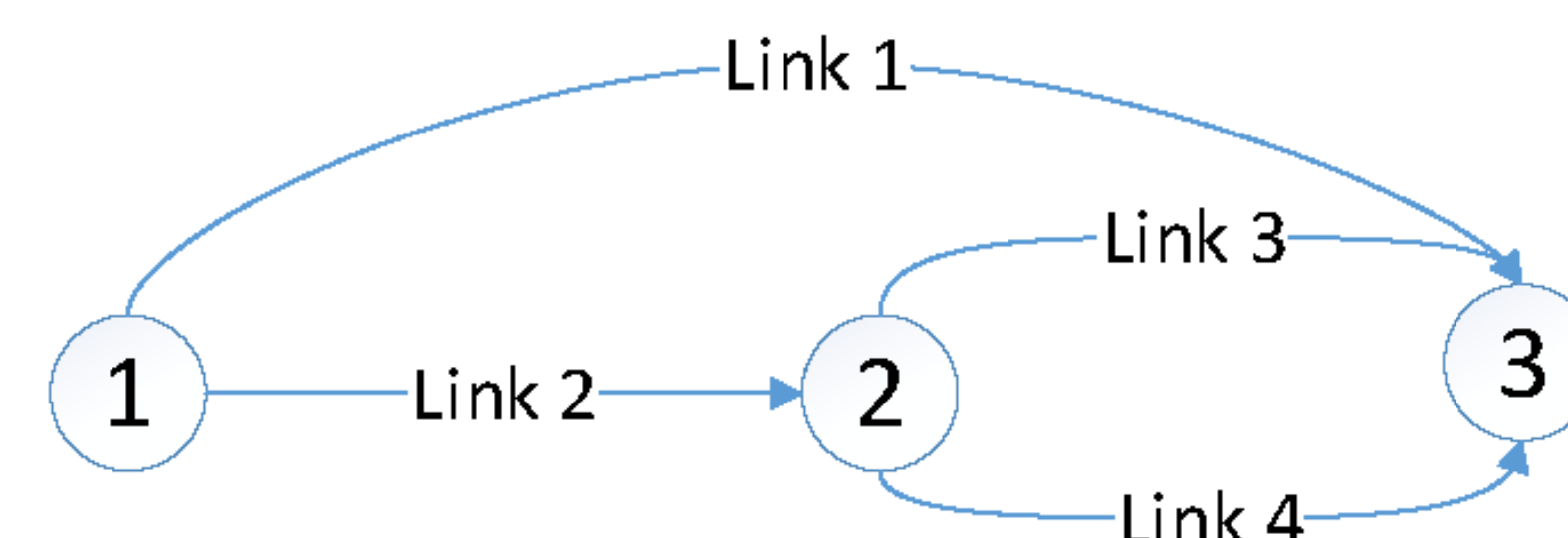
### Multilevel structure

Level 1:	$X_m \sim N(X + e, \Sigma_x + \Sigma_e)$	Measurement error
Level 2:	$X \sim N(\Delta p Q, \Sigma_x)$ $F \sim N(p Q, \Sigma_f)$	Route choice variation
Level 3:	$Q \sim N(q, \Sigma_q)$	OD variation

## Experiment results

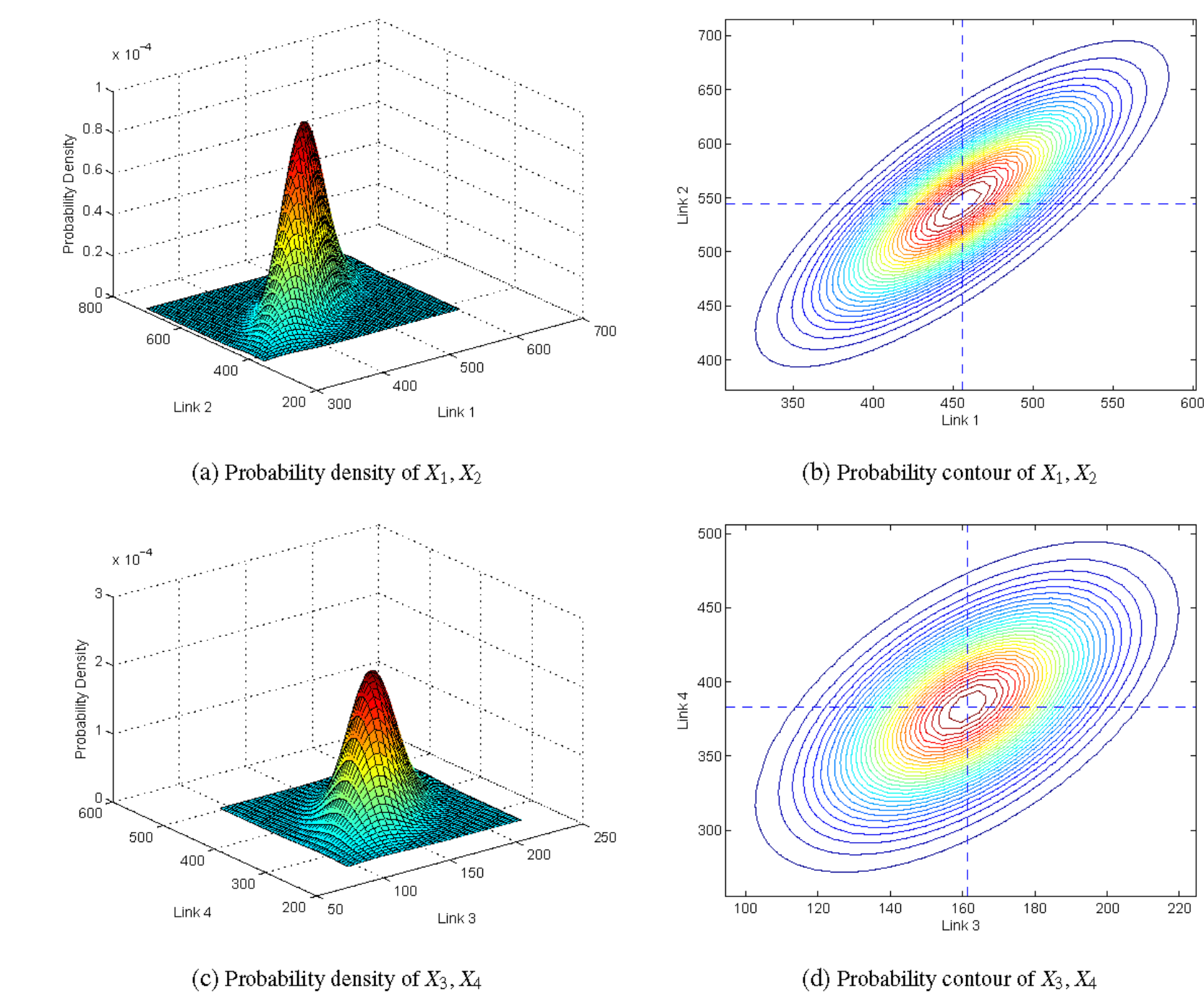
- We perform the GESTA on a small network as follows, the analytical properties of link/path flows, travel costs can be derived.

### Small network: 3-path network



- BPR link travel time function is adopted,  $q = 1000, \sigma_{OD}^2 = 10000, \sigma_e^2 = 100, cap_a = 360$

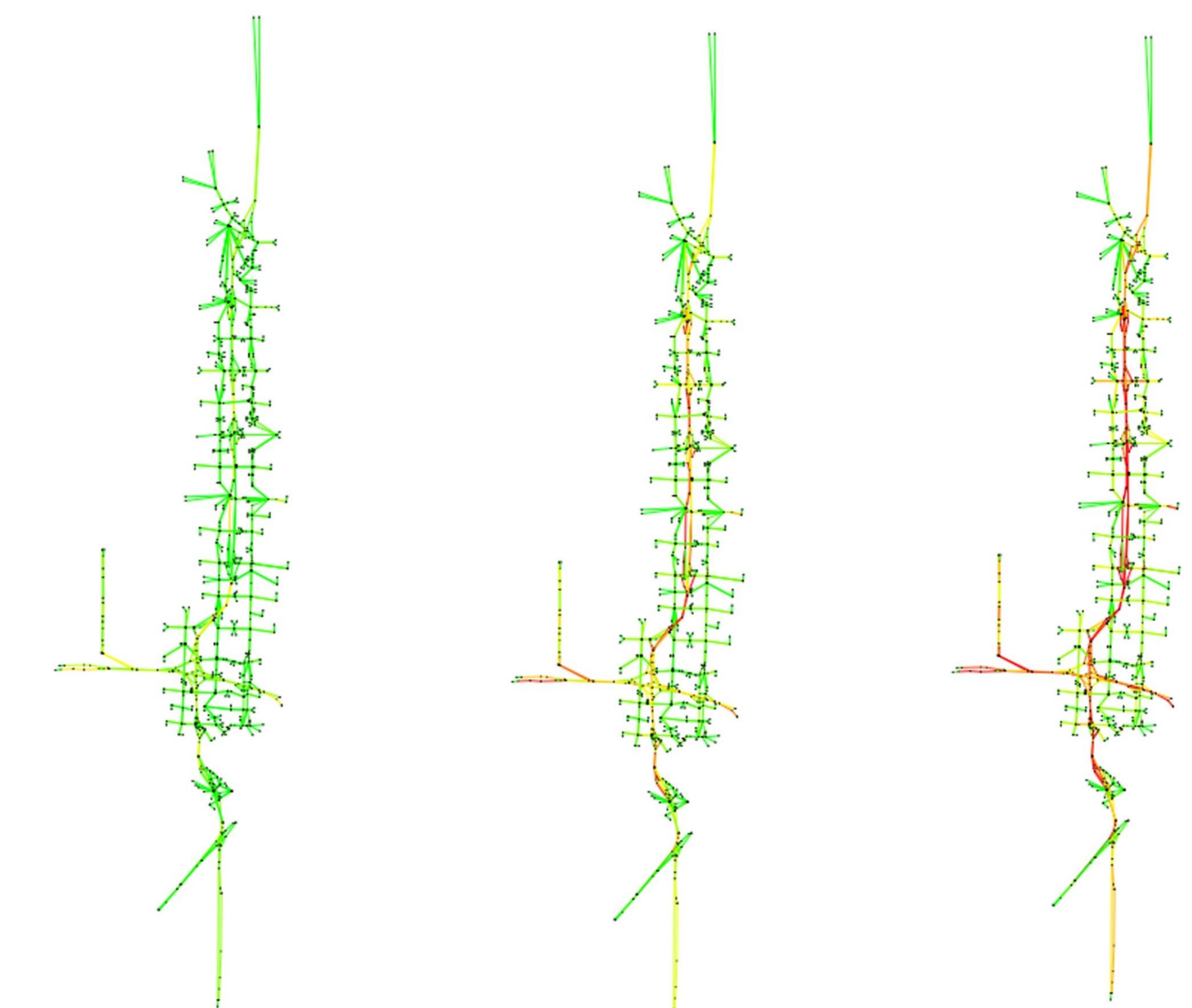
- The following figures are the probability density and contour of  $X_1$  and  $X_2, X_3$  and  $X_4$ . The contours can also be interpreted as confidence intervals under different confidence levels.



- Comparison with classical assignment models

Links	GESTA	Logit-SUE, $\Theta = 0.1$	Logit-SUE, $\Theta = 0.01$	UE
Link 1	$\mathcal{N}(455.6, 49.2^2)$	422.3	360.1	410.4
Link 2	$\mathcal{N}(544.4, 57.5^2)$	577.7	639.9	589.6
Link 3	$\mathcal{N}(161.4, 22.3^2)$	263.8	316.5	132.6
Link 4	$\mathcal{N}(383.0, 42.5^2)$	313.9	323.4	457.0

### Large network: SR41 corridor



The 5%, 50% and 95% quantiles link flows (Using Rectangle CI approximation. Red represents volume/capacity 1, and green represents volume/capacity=0, other colors are smoothly transitioned from green to red as volume/capacity increases from 0 to 1)