Abstract
Origin-Destination (OD) demand is an indispensable component for modeling transportation networks. A bi-level optimization approach considering equilibrium constraints is computationally challenging for large-scale networks, which prevents the OD estimation (ODE) being scalable. To solve for ODE in large-scale networks, this paper develops a generalized single-level formulation for ODE incorporating Stochastic User Equilibrium (SUE) constraints. Two single-level ODE models are specifically discussed and tested. One employs an SUE based on the satisfaction function, and the other is based on the Logit model. Analytical properties of the new formulation are analyzed. Gradient-based algorithms are proposed to solve for this formulation. Numerical experiments are conducted on a small network and a large network, along with sensitivity analysis on sensor location, historical OD information and measurement error. Results indicate that the new single-level formulation, in conjunction with the proposed solution algorithms, can achieve comparable accuracy as the bi-level formulation, while being more computationally efficient for large networks.

Background
Traditional bi-level ODE uses its upper-level formulation to minimize the square error between estimated and observed traffic counts.
• The uniqueness of the estimated OD demand is usually not guaranteed.
• The computational complexity of the bi-level programming.
• How to incorporate SUE constraints into a single-level framework has not been explored.

Contributions
• It proposes a generalized form for single-level ODE formulation with general equilibrium constraints and revisits each term in the formulation.
• It derives two specific SUE based ODE formulations and proves the unbiasedness of the OD estimator.
• The proposed methods are tested on a large-scale real network to gain insights from solutions. The computational efficiency of the solution algorithms is also examined.

Framework
The generalized ODE formulation with equilibrium constraint can be formulated as follows:
\[
\min \epsilon_m + w_e (\epsilon_e + \kappa) \\
\text{s.t. } (q, x, f) \in H
\]
where
\[
\epsilon_m = \frac{1}{2} \sum_{a \in A} (x_a - x_d^a)^2 + \frac{w_d}{2} \sum_{rs \in K_d} (q_{rs} - q^h_{rs})^2
\]

• Satisfaction function based SUE model
\[
\epsilon_e = -\sum_{rs} q_{rs} w_{rs} c_{rs}(x) + \sum_{a} x_a t_a(x_a) - \sum_{a} \int_0^x t_a(\omega)d\omega
\]

• Logit-based SUE model
\[
\epsilon_e = \sum_{a \in A} \int_{0}^{x_a} t_a(w)dw + \frac{1}{\Theta} \sum_{rs \in W} \sum_{k \in K_{rs}} f_{rs}^{k} \log f_{rs}^{k}
\]

Solution Algorithm
The solution procedure is summarized as follows:
• Step 0: (Initialization). Iteration i = 1, generate a path set for each O-D pair. Set path flow \( r_{rs}^{k}(i) \) and link flow \( x_a^{(i)} \) as zero.
• Step 1: (Path cost update). Compute \( r_{rs}^{k}(i) \) by the new network conditions \( x_a^{(i)} \) and \( f^{k}(i) \).
• Step 2: (Update adjustment factor). If using satisfaction function based formulation, skip; if using logit based formulation, use Equation (26) to update adjustment factor \( \kappa \).
• Step 3: (Gradient descent). Perform a one-step gradient projection descent or Frank–Wolfe descent.
• Step 4: (Network Update). Update path flow \( r_{rs}^{k}(i) \) and link flow \( x_a^{(i)} \).
• Step 5: (Convergence check). Check the difference of link flow if, if the convergence criterion is met, go to Step 6; if not, i = i + 1, go to Step 1.
• Step 6: (Output). Output (f, q).