A Generalized Single-level Formulation for Origin-Destination Estimation under Stochastic User Equilibrium

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Abstract

Origin-Destination (OD) demand is an indispensable component for modeling transportation networks. A bi-level optimization approach considering equilibrium constraints is computationally challenging for large-scale networks, which prevents the OD estimation (ODE) being scalable. To solve for ODE in large-scale networks, this paper develops a generalized single-level formulation for ODE incorporating Stochastic User Equilibrium (SUE) constraints. Two single-level ODE models are specifically discussed and tested. One employs an SUE based on the satisfaction function, and the other is based on the Logit model. Analytical properties of the new formulation are analyzed. Gradient-based algorithms are proposed to solve for this formulation. Numerical experiments are conducted on a small network and a large network, along with sensitivity analysis on sensor location, historical OD information and measurement error. Results indicate that the new single-level formulation, in conjunction with the proposed solution algorithms, can achieve comparable accuracy as the bi-level formulation, while being much more computationally efficient for large networks.

Background

Traditional bi-level ODE uses its upper-level formulation to minimize the square error between estimated and observed traffic counts.

- The uniqueness of the estimated OD demand is usually not guaranteed.
- The computational complexity of the bi-level programming.
- **How to incorporate SUE constraints into a single-level framework has not been explored.**

Contributions

- It proposes a generalized form for single-level ODE formulation with general equilibrium constraints and revisits each term in the formulation.
- It derives two specific SUE based ODE formulations and proves the unbiasedness of the OD estimator.
- The proposed methods are tested on a large-scale real network to gain insights from solutions. The computational efficiency of the solution algorithms is also examined.

Framework

The generalized ODE formulation with equilibrium constraint can be formulated as follows:

$$\min_{q,x,f} \epsilon_m + w_e(\epsilon_e + \kappa)$$
$$\text{s.t.} \quad (q,x,f) \in H$$

where

$$\epsilon_m = \frac{1}{2}\sum_{a\in A^o}(x_a - x_a^o)^2 + \frac{w_q}{2}\sum_{rs\in K_q}(q_{rs} - q_{rs}^H)^2$$

❖ Satisfaction function based SUE model

$$\epsilon_e = -\sum_{rs} q_{rs} S_{rs}[c^{rs}(x)] + \sum_a x_a t_a(x_a) - \sum_a \int_0^{x_a} t_a(\omega)d\omega$$

❖ Logit-based SUE model

$$\epsilon_e = \sum_{a\in A}\int_0^{x_a} t_a(w)dw + \frac{1}{\Theta}\sum_{rs\in W}\sum_{k\in K_{rs}} f_{rs}^k \log f_{rs}^k$$

Solution Algorithm

The solution procedure is summarized as follows:

- **Step 0**: *(Initialization).* Iteration $i = 1$, generate a path set for each O-D pair. Set path flow $f_{rs}^{k\,(i)}$ and link flow $x_a^{(i)}$ as zero.
- **Step 1**: *(Path cost update).* Compute $c_{rs}^{k\,(i)}$ by the new network conditions $x_a^{(i)}$ and $f_{rs}^{k\,(i)}$.
- **Step 2:** *(Update adjustment factor).* If using satisfaction function based formulation, skip; if using logit based formulation, use Equation (26) to update adjustment factor κ.
- **Step 3:** *(Gradient descent).* Perform a one-step gradient projection descent or Frank–Wolfe descent.
- **Step 4:** *(Network Update).* Update path flow $f_{rs}^{k\,(i)}$ and link flow $x_a^{(i)}$.
- **Step 5:** *(Convergence check).* Check the difference of link flow f, if the convergence criterion is met, go to Step 6; if not, $i = i + 1$, go to Step 1.
- **Step 6:** *(Output).* Output (f, q).

Numerical Experiments

❖ A small network

$$t_a(X_a) = t_a^0\left[1 + \alpha\left(\frac{X_a}{cap_a}\right)^\beta\right]$$

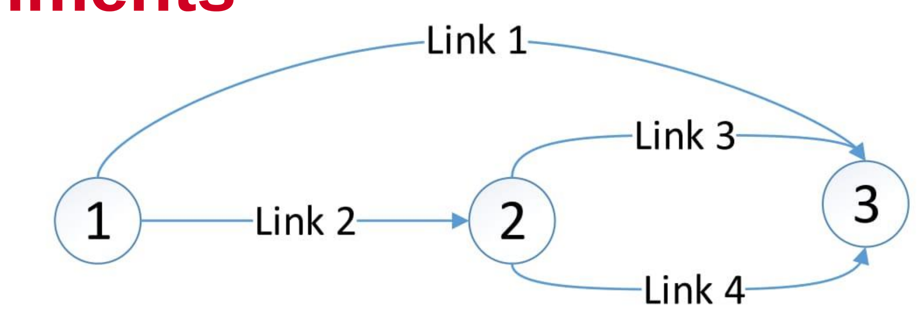
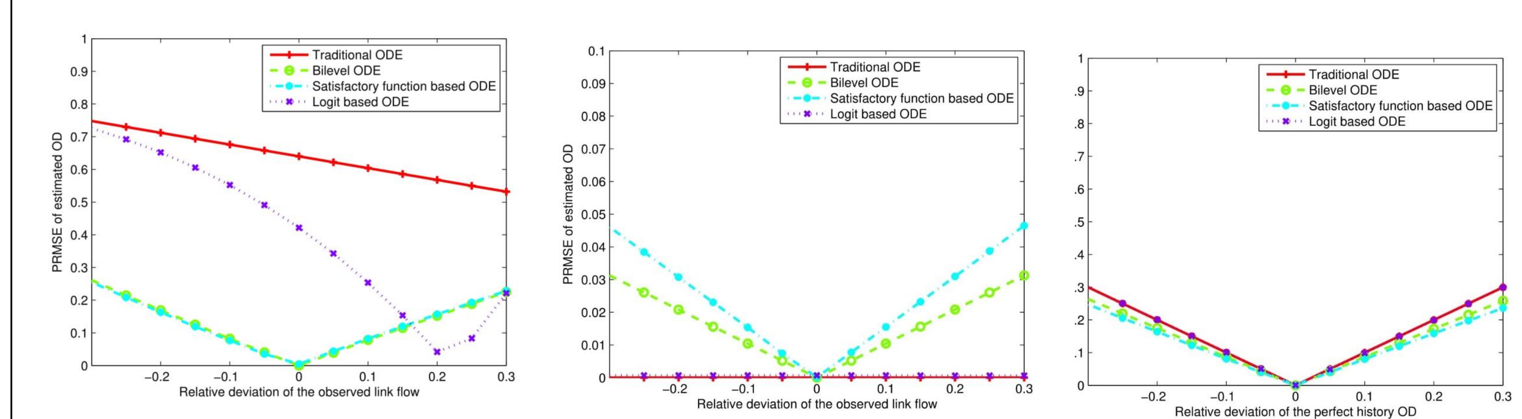


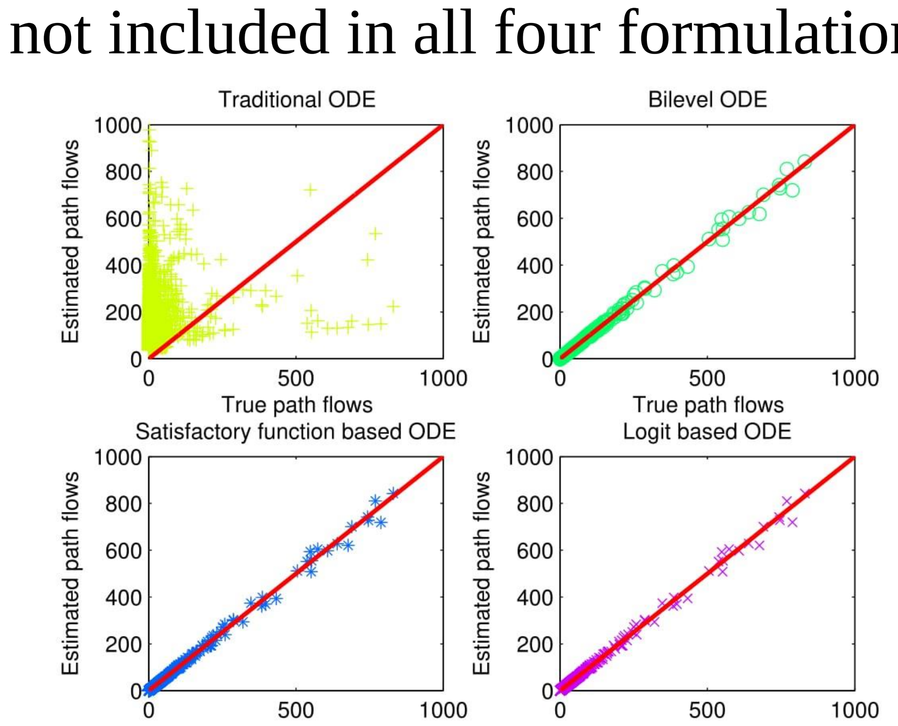
Figure 1 A three-path small network

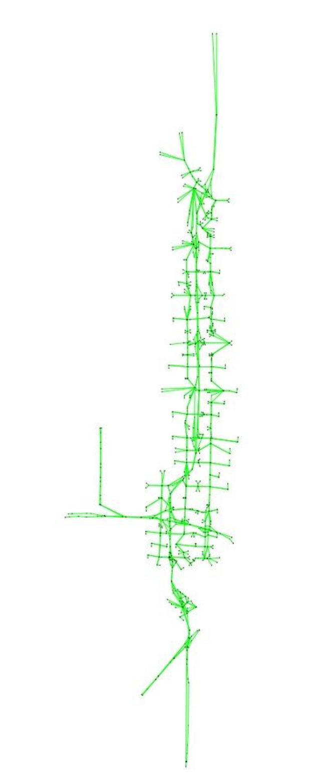
	Observed link number	Estimated link flows				Estimated OD metrics		
		Link 1	Link 2	Link 3	Link 4	PRMSE (%)	MARE (%)	PTD (%)
Bi-level ODE	1	360	640	316	324	0.02	0.02	0.02
	2	360	640	317	323	0.01	0.01	0.01
	3	360	640	316	324	0.05	0.05	0.05
	4	360	640	316	324	0.05	0.05	0.05
	1,2	360	640	320	320	0.01	0.01	0.01
	1,3	360	640	317	323	0.01	0.01	0.01
	1,4	360	640	317	323	0.01	0.01	0.01
Satisfaction function based ODE	1	362	643	318	325	0.48	0.48	-0.48
	2	393	663	328	335	5.58	5.58	-5.58
	3	317	591	297	294	9.22	9.22	9.22
	4	319	594	289	305	8.79	8.79	8.79
	1,2	360	640	317	223	0.15	0.15	0.15
	1,3	361	642	317	325	0.31	0.31	-0.31
	1,4	361	642	318	324	0.30	0.30	-0.30
Logit based ODE	1	360	399	199	200	24.17	24.17	24.17
	2	1	639	320	319	35.95	35.95	35.95
	3	109	358	316	42	53.25	53.25	53.25
	4	119	367	44	323	51.38	51.38	51.38
	1,2	360	640	320	320	0.12	0.12	0.12
	1,3	360	446	316	130	19.43	19.43	19.43
	1,4	360	465	142	323	17.58	17.58	17.58



❖ SR-41 corridor

We randomly pick 6% of links to be observable and the observations are free of noise, historical OD information is not included in all four formulations.





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